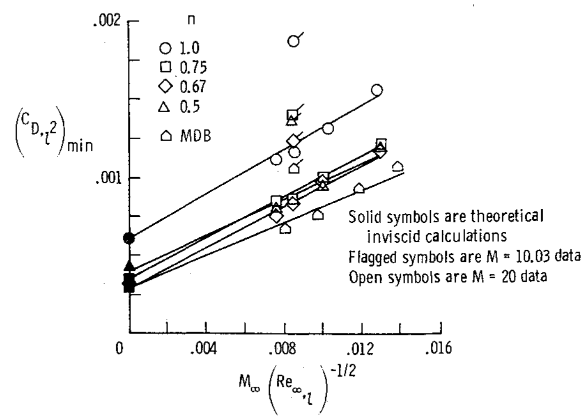

Fig. 2 Minimum drag coefficient variation as a function of $M_\infty(Re_{\infty,l})^{-1/2}$.

A better correlation parameter for the present data is the "hypersonic viscous parameter," $\bar{V}'_\infty = M_\infty(C'_\infty/Re_{\infty,l})^{1/2}$, which was shown by Whitfield and Griffith[4] to be useful in correlating hypersonic zero-lift drag data on cones with theory. Values of $\bar{V}'_\infty$ for the data were calculated from test conditions using the equations from Ref. 5 to determine the C'_∞. These equations are formulated such that for helium the variation of viscosity with temperature follows a power law, while for air a modified form of Sutherland's Law is followed. Monaghan's relationship was used to determine reference temperature. The correlation with $\bar{V}'_\infty$ of the zero-lift drag coefficient, based on body length squared, is shown in Fig. 3. The correlation is excellent, considering that it not only involves inviscid and viscous conditions, varying Mach number, and ratio of specific heats, but it also retains linearity.

Conclusions

Results presented support use of the hypersonic viscous parameter, $\bar{V}'_\infty$, advocated by Whitfield and Griffith in the correlation of hypersonic zero-lift drag results obtained from a wide spectrum of state conditions varying from the theoretical inviscid conditions to viscous flow. The excellent correlation obtained for zero-lift drag data of a series of power-law bodies and a theoretical minimum drag body offers promise that this parameter may also be useful in extrapolating ground facility data to flight conditions. Additional wind-tunnel tests at different conditions and free-flight data are needed to verify this conclusion.

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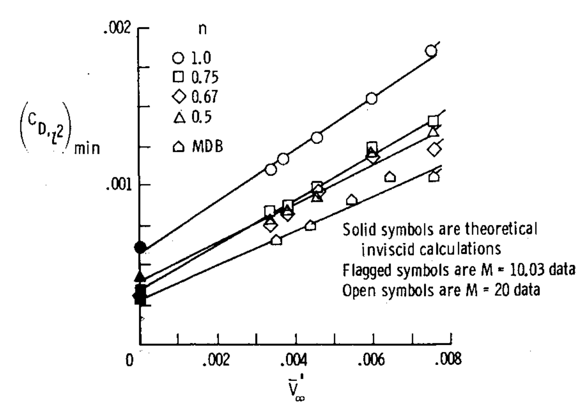

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Correlation of Ion and Beam Current Densities in Kaufman Thrusters

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IN the absence of significant direct impingement erosion, electrostatic thruster accelerator grid lifetime will be determined by the charge exchange erosion that occurs where the ion beam current density peaks. In order to maximize the thrust from an engine with a specified grid lifetime, the ion beam current density profile should therefore be as flat as possible. Knauer[1] has suggested this can be achieved by establishing a radial plasma uniformity within the thruster discharge chamber, and his tests with the radial field thruster provide an example of uniform plasma properties within the chamber and a flat ion beam profile occurring together. The following study shows that in particular the ion density profile within the chamber determines the beam current density profile and that a uniform ion density profile at the screen grid end of the discharge chamber should lead to a flat beam current density profile.

A 15 cm, SERT II thruster described in Ref. 2 was equipped alternately with flat grids (0.23 cm separation) and dished grids (0.089 cm separation) and was operated over the range of beam current and high voltage conditions indicated in Table 1. In addition the cathode pole piece was modified so the critical magnetic field line and hence the ion density profile at the screen grid could be altered. The mean ion beam current density profile at the accelerator grid was determined by sweeping a Faraday probe through the beam at the four axial locations indicated in Fig. 1 and extrapolating the resulting data at several radial positions. back to the plane tangent to the center of the accel grid. Similarly, Langmuir probe data were obtained at locations shown on Fig. 1 which are 11 mm and 23 mm from the plane of contact of the screen grid and thruster body (Tests 1–5) or the 11 mm, 17 mm and 23 mm locations (Tests 6–9). These data were then used to extrapolate to the ion densities at this plane of contact. The current and ion density profiles at the accel and screen grids were normalized using, respectively, the maximum current and ion densities corresponding to the profile in question.

A typical comparison between the resulting profiles is presented as Fig. 2 (Test 3 of Table 1). A linear regression analysis[3] was conducted using the ion density data points shown as solid symbols on the typical plot and the values of beam current density at the same radial location (picked from the dotted line). The resulting correlation coefficients for each set of data, as well as the correlation coefficient for the pooled data (0.945) are given in Table 1. Since the data were

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Table 1 15 cm thruster operation conditions for beam current—density correlation study

Test no.	Beam current	High voltage pos.	High voltage neg.	Thruster configuration	Correlation coefficient[a, 3]	Current density ratio
1	0.354 amp	3.4 kv	−1.7 kv	SERT II	0.971	1.06
2	0.171 amp	3.4 kv	−1.7 kv	SERT II	0.904	1.83
3	0.148 amp	1.5 kv	−0.5 kv	SERT II	0.911	1.21
4	0.235 amp	3 kv	−1.5 kv	SERT II	0.983	1.53
5	0.178 amp	3.4 kv	−1.7 kv	SERT II	0.971	1.22
6	0.154 amp	3.4 kv	−1.7 kv	2-cm-diam cathode pole piece	0.970	1.24
7	0.356 amp	3.4 kv	−1.7 kv	2-cm-diam cathode pole piece	0.988	1.41
8	0.071 amp	1 kv	−0.5 kv	2-cm-diam pole piece dished grids	0.963	0.90
9	0.128 amp	1 kv	−0.5 kv	2-cm-diam pole piece dished grids	0.964	0.95

[a] Correlation coefficient for all data = 0.945.

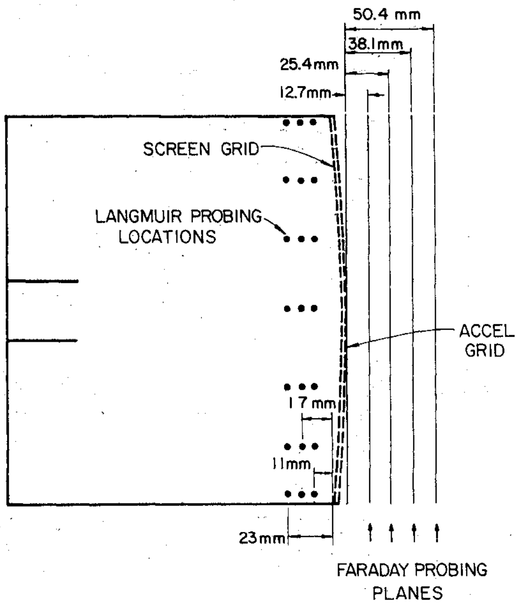

Fig. 1 Probing locations.

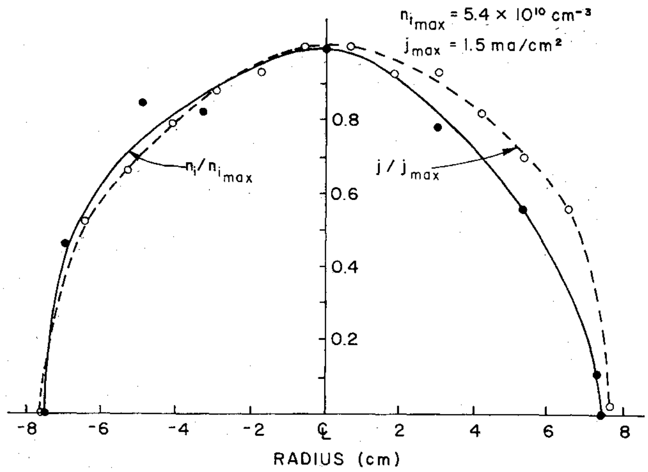

Fig. 2 Typical shape correlation—Ion beam current density and ion density at screen grid.

collected over a range of flow distribution, beam current and high-voltage conditions and the correlation remained high, there is a high probability that beam current profile is determined by the ion number density profile and is dependent on other thruster parameters only insofar as they affect this profile.

Tests 2 and 5 which were conducted at essentially the same conditions provide an indication of the variation in correlation coefficient which might be expected in the study. The variation is considered to be due primarily to errors in Langmuir probe data evaluation.

The correlation between magnitudes of the beam current and ion densities for each similar profile pair is indicated in the last column of Table 1 which presents the ratio of centerline current density in the ion beam to centerline current density of ions approaching the screen grid from within the thruster. The ion current density within the thruster was calculated using the modified Bohm criterion[4] on the basis that it determines the minimum ion velocity for a stable sheath at the screen grid. Values of this ratio calculated from flat grid data show the average current density in the ion beam is greater than that in the thruster. This suggests ions are either drawn at a velocity above the minimum velocity for a stable sheath or they are drawn toward the screen grid from an irregular surface in the thruster having a surface area greater than the cross sectional area of the ion beam. When dished grids are employed and values of the ratio fall below unity an extraction area within the thruster which is less than that of the ion beam is implied.

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